

Problem Solving

{ Dr. Melanie Maxwell & Dr. Steven Sher
MaTHink, March 30, 2013

Today you will be able to:

- ⌘ Deepen your understanding of **problem solving**.
- ⌘ Identify and apply stages of the problem solving **process**
- ⌘ Identify and apply a variety of problem solving **strategies**.
- ⌘ Discuss and begin **plan** for implementing instruction to increase student's problem skills and aptitude.

Objectives



Problem Solving & Current Reality

{ How do your students currently
view problem solving?

S: I don't get this problem.

M: Why?

S: There is a remainder and there isn't supposed to be a remainder.

M: How do you know?

S: Because the rest of our problems didn't have a remainder.

M: Could this problem be different?

S: No, that is not how we were shown to solve these.

Does this sound familiar?

It gets better.

M: What does the 351 mean?

S: Look at my paper, it shows that I multiplied.

M: Yes, but what does it mean?

S: What do you mean?

M: Remember when you learned to multiply in 3rd grade and you made arrays? Lets try a simpler problem to show this.

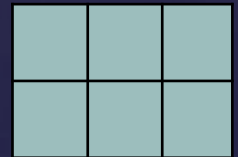
S: (grumbling)

M: Each square is 1 foot by 1 foot. So it is a square foot.

S: Yes, but that doesn't help me.

M: So if each square foot is \$8 (wrote \$8 in each square), how much would it cost.

S: But that is not how my teacher did it.



I had to walk away at this point.

I am the adult. I am the adult. . .

M: Will you consider that this problem may be different than the ones your teacher showed you?

S: Ok

M: Lets check what you understand. Does it make sense that the 27 X 13 equals 351 squares?

S: Yes.

M: Now if each square is \$8, how much will it cost?

S: $8 \times \text{what} = 351$?

M: Why?

S: Reread the entire problem aloud.

M: What does that mean?

S: (silence)

M: How many squares do you you have?

S: 351, oh now I get it.

M: So it was different than the other problems.

S: What if you are wrong?

Problem Solving Desired Reality

{ How do we want students to view
problem solving?

CCSS MP Goals

Common Core State Standards Mathematical Practices

& Shift the paradigm **from** algorithms for isolated procedures **towards** flexible skills needed for real world problem solving (MP 4- Modeling)

& Math to meet future problems

Isolated and “Inert Ideas”



Alfred North Whitehead, *Aims of Education* (1929, p. 4)

Coherent and Useful Knowledge



Problem Solving Standard

MP 1 Make sense of problems and
persevere in solving them...

Students start by explaining to themselves
the meaning of a problem and looking for
entry points to its solution.

Problem Solving and Making Sense

What is making sense?

Student outcome – know and do

How will learning occur?

The activities students must engage in

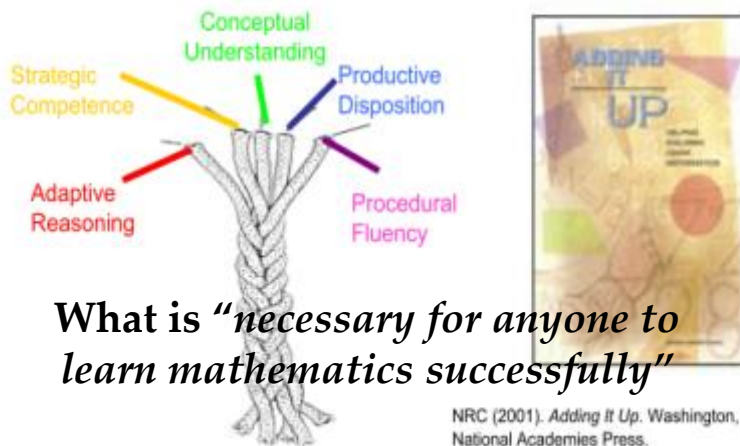
CCSS Mathematical Practices

NRC Mathematical Proficiencies
(Adding It Up, 2001)

NCTM Process Standards (2000)

Underlying Frameworks

Strands of Mathematical Proficiency

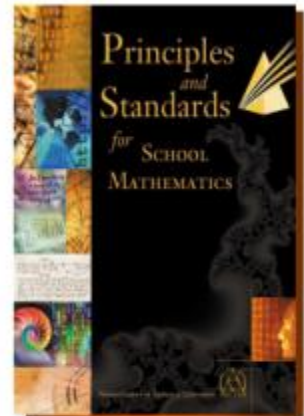


Underlying Frameworks

National Council of Teachers of Mathematics

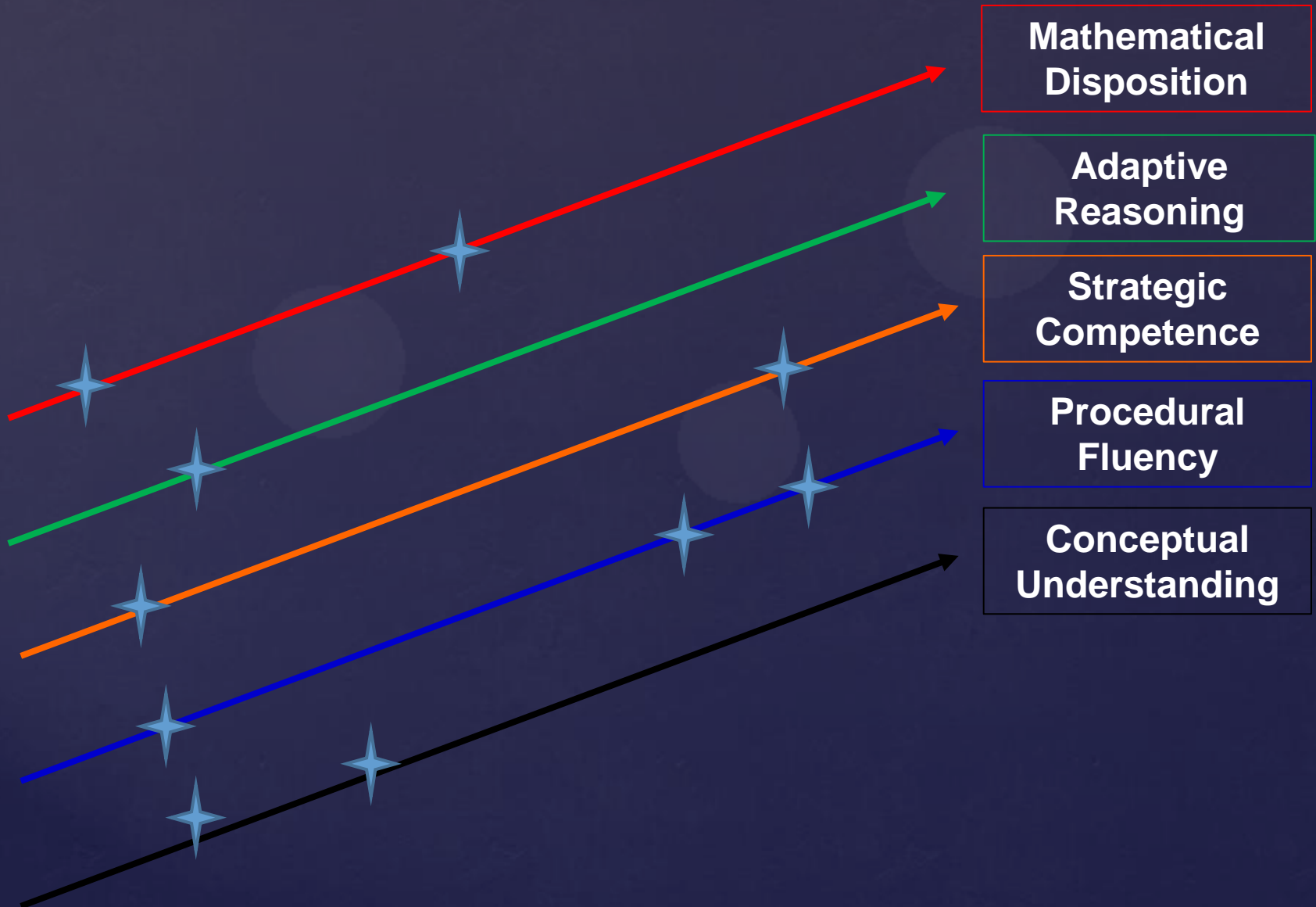
5 **Process** Standards

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representations



NCTM (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

Strands of Proficiency - Continua of Learning

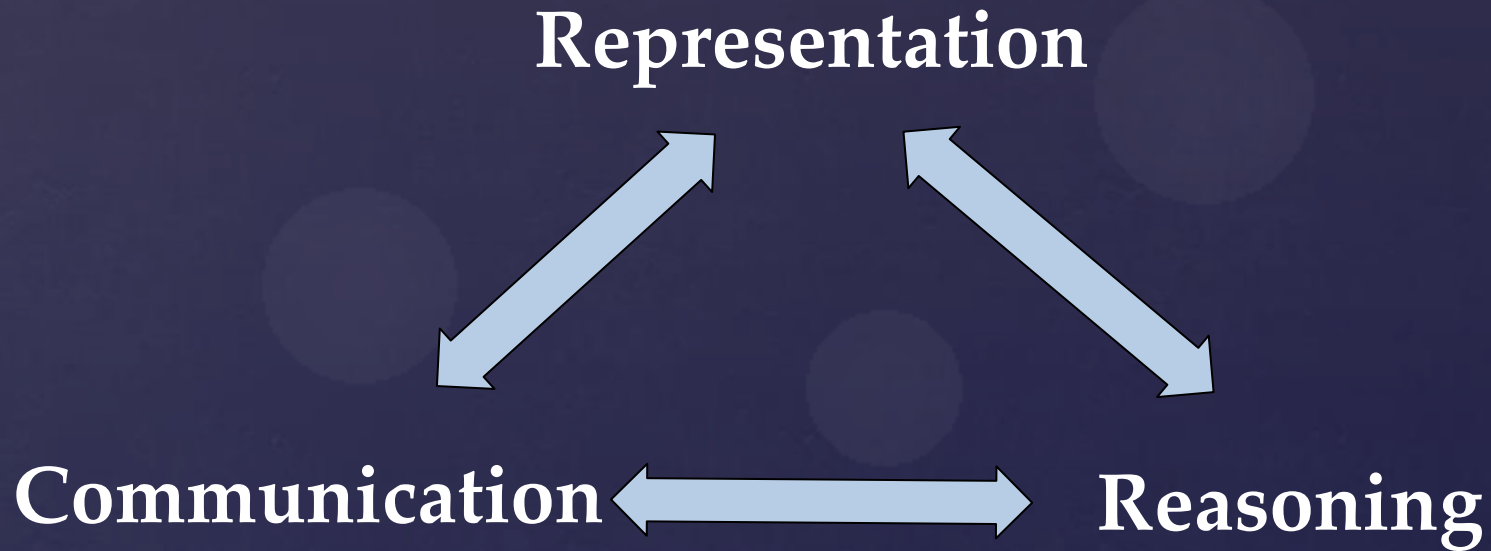


NCTM Process Standards

Problem Solving	Reasoning and Proof	Communication	Connections	Representations
Build new knowledge	Fundamental Aspect of math	Organize and consolidate thinking	Use connections between math. ideas	Create and use representations to organize, record, and communicate math. ideas
Solve problems From a variety Of contexts	Make and investigate conjectures	Communicate coherently and clearly to peers, and others	Math. ideas interconnect and build into a coherent whole	Select, apply, translate between math. representations to solve problems
Apply a variety of Strategies	Develop and evaluate math. conjectures	Analyze and evaluate the thinking of others	Recognize and apply math. in contexts outside of math.	
Monitor and Reflect on the Process	Select and use various types of reasoning and methods of proof	Use the language to express math. ideas precisely		Model and interpret physical, social, and math phenomena

Making Sense

-the first step in Problem Solving



Making **Connections** within and outside of mathematics

How do we bridge
this gap?
{ Now!

Today you will be able to:

- ⌘ Deepen your understanding of **problem solving**.
- ⌘ Identify and apply stages of the problem solving **process**
- ⌘ Identify and apply a variety of problem solving **strategies**.
- ⌘ Discuss and begin **plan** for implementing instruction to increase student's problem skills and aptitude.

Objectives



George Pólya

Understand the Problem

Devise a Plan

Carry out the Plan

Look Back


Activity:

Read Standard for Mathematical Practice 1 (SMP 1).

Identify correlations between Pólya's research and SMP 1.


Problem Solving Process

Problem Solving Strategies

Strategy	Example	When I Use It
Make a Table	Phil and Marcy spent all day Saturday at the fair. Phil rode 3 rides each half hour and Marcy rode 2 rides each half hour. How many rides had Marcy ridden when Phil rode 24 rides? 	Try making a table where: • there are 2 or more quantities, • amounts change using a pattern.


Strategy	Example	When I Use It
Act It Out	How many ways can 3 students shake each other's hand? 	Think about acting out a problem when the numbers are small and there is action in the problem you can do.


Strategy	Example	When I Use It
Draw a Picture	The race was 5 kilometers. Markers were at the starting line and the finish line. Markers showed each kilometer of the race. Find the number of markers used. 	Try drawing a picture when it helps you visualize the problem or when the relationships such as joining or separating are involved.


Strategy	Example	When I Use It
Work Backward	Tracy has band practice at 10:15 A.M. It takes her 20 minutes to get from home to practice and 5 minutes to warm up. What time should she leave home to get to practice on time? 	Try working backward when: • you know the end result of a series of steps, • you want to know what happened at the beginning.


Strategy	Example	When I Use It
Make a Graph	Marcy was in a jump rope contest. How did her number of jumps change over the five days of the contest? 	Make a graph when: • data for an event are given, • the question can be answered by reading the graph.

Strategy	Example	When I Use It
Write an Equation	Maria's new CD player can hold 6 discs at a time. If she has 204 CDs, how many times can the player be filled without repeating a CD? Find $204 \div 6 = n$.	Write an equation when the story describes a situation that uses an operation or operations.

Strategy	Example	When I Use It
Try, Check, Revise	Suzanne spent \$27, not including tax, on dog supplies. She bought two of one item and one of another item. What did she buy? $\$6 + \$6 + \$15 = \27 $\$7 + \$7 + \$12 = \26 $\$6 + \$6 + \$15 = \27 	Use Try, Check, Revise when quantities are being combined to find a total, but you don't know which quantities.

Strategy	Example	When I Use It
Look for a Pattern	The house numbers on Forest Road change in a planned way. Describe the pattern. Tell what the next two house numbers should be. 	Look for a pattern when something repeats in a predictable way.

Strategy	Example	When I Use It
Solve a Simpler Problem	Each side of each triangle in the figure at the left is one centimeter. If there are 12 triangles in a row, what is the perimeter of the figure? I can look at 1 triangle, then 2 triangles, then 3 triangles.  perimeter = 3 cm perimeter = 4 cm perimeter = 5 cm	Try solving a simpler problem when you can create a simpler case that is easier to solve.

Strategy	Example	When I Use It
Make an Organized List	How many ways can you make change for a quarter using dimes and nickels? 	Make an organized list when asked to find combinations of two or more items.

PS Strategies **Activity I**

Task: Solve the problem using the PS strategy assigned to your team.

Record your work, solution, and answers to the focus questions on your chart.

A Web site that rents DVDs charges a one-time fee of \$7 to become a member of the site. It costs \$3 to rent each DVD. Diana joined the site and spent a total of \$61. How many DVDs did she rent?

Focus Questions (Metacognition)

- ◉ How did you start this strategy?
- ◉ What challenges did you have applying this strategy?
- ◉ What are the advantages of your strategy for this problem?
- ◉ What are the disadvantages of your strategy for this problem?
- ◉ How does this strategy clarify the mathematics in this problem?
- ◉ What did you need to understand to be able to apply your strategy?

- ⌘ Travel as a group with your poster.
- ⌘ 3 minute sharing with at least 3 other groups.
 - ⌘ Two teams pair up and position posters so all team members can see.
 - ⌘ Form partners (dyads or triads) across teams.
 - ⌘ Choose one focus question to discuss.
 - ⌘ Discuss the focus question with insights from each team. (No monopolizing the discussion.)
 - ⌘ Rotate, repeat, pick a different focus question.
- ⌘ Write one sentence summarizing an “aha!”
- ⌘ Whole group whip around, biggest aha!

Debriefing

- ⌘ **Sort problems** according to different strategies you would use.
- ⌘ **Explain** what about the problem made you think to use that strategy.
- ⌘ Goal: Articulate the why and how specific strategies help with certain problems.

PS Strategies **Activity II**

Teaching Problem Solving Skills

{ How do we create opportunities for students to develop process and strategy skills?

- ⌘ Solve using _____ strategy then discuss focus questions.
- ⌘ Problem Sorts
- ⌘ Explicit direct instruction of PS strategies. (How does it work?)
- ⌘ Consistent use of Think Aloud
- ⌘ Labeling processes and strategies students use
- ⌘ Discussing possible strategies (which ones, why, how will you start) before solving
- ⌘ Sample problems for students to reference, “It is like the monkey problem.”

Ways to build PS skills

Problem Solving Process

Read and Understand

K

W

C

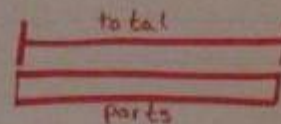
- What do I know?
- Reread the problem.

What am I trying to find?
I need to know...

- Are there any:
- special condition?
- connections I can make?

Plan and Solve

- Choose a PS Strategy
- **Try a Bar Diagram**
- Answer the Question



Look Back and Check

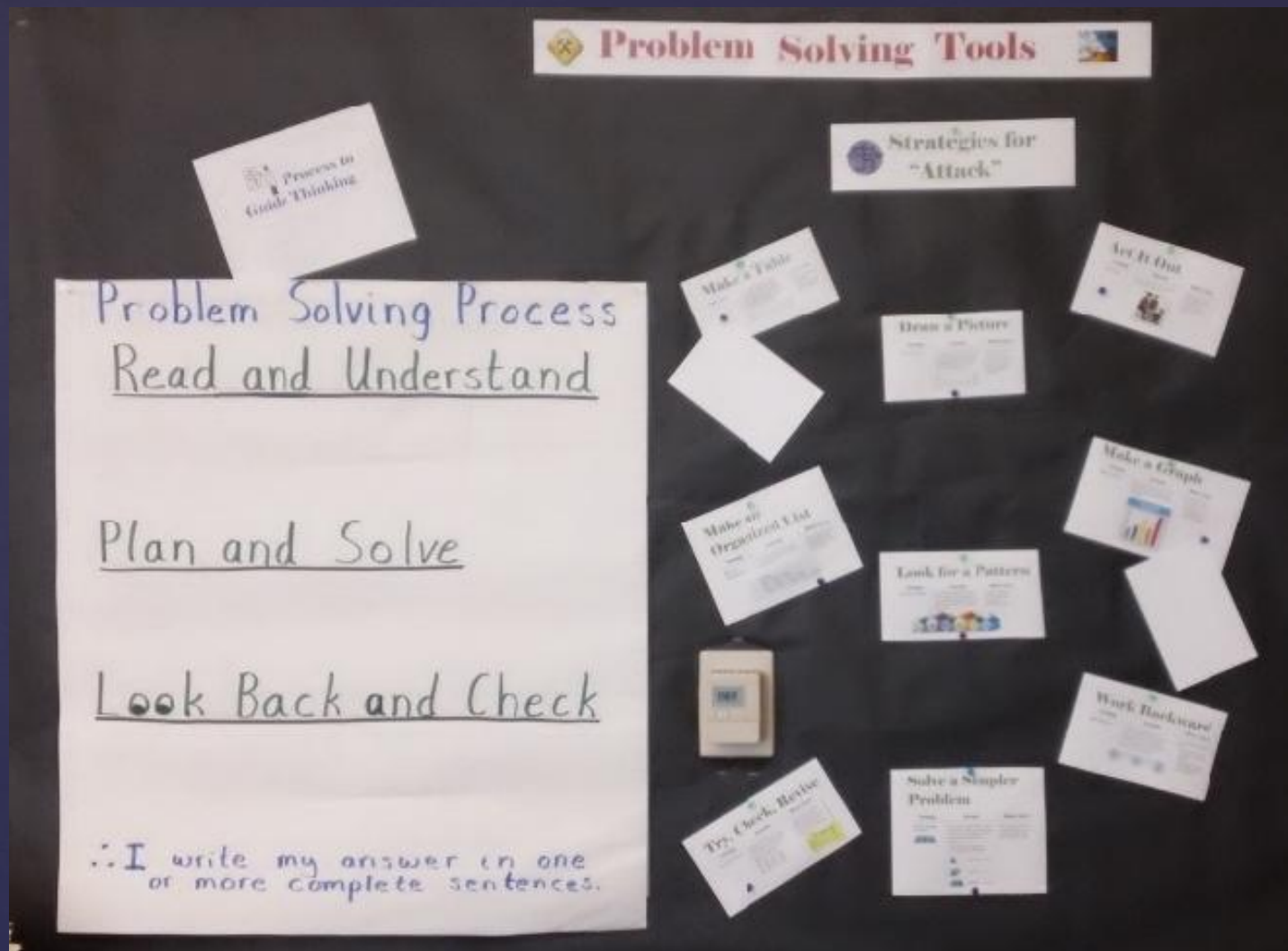
- Did I answer the question?
- Is my answer reasonable?
 - Estimate - Rounding or Compatible Numbers
 - Use Inverse operations - $+$ \leftrightarrow $-$ \times \leftrightarrow \div

\therefore I write my answer in a complete sentence.



-----?-----	
23	18

-----24-----			
?	?	?	3 times as many
?			



One Method – PS Wall

⌘ Climate is essential

⌘ Be less helpful, but be very supportive.

Building PS aptitude

⌘ Problem solving is . . .

⌘ What do you plan to implement to begin moving from your current reality to a CC reality of students as engaged problem solvers?

⌘ What can you explore THIS year?

⌘ How will you begin next year? (KISS)

Summary & Action Plan



{ On behalf of Samantha,
Michael, Melissa,
Angelique, Ruth, . . .

Thank You!

